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Three-dimensional instability in the wake of a pitching foil

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Abstract

In this paper, we investigate the three-dimensional transition of a propulsive wake, which is generated by a pitching foil, mimicking the caudal fin of a fish. We note that a fish fin is usually of very short aspect ratio, here we assume an infinite span, aiming to reveal the fundamental physical mechanisms. For the base flow, three sequential wake structures are identified as we increase the pitching amplitude, while fixing the pitching frequency. They are Bénard-von Kármán (BvK) vortex street, reverse BvK vortex street and deflected wake, respectively. Through Floquet analysis, we observe an unstable mode at the wavelength $\lambda_z = 0.21$, while another mode also becomes critical with a longer wavelength at $\lambda_z = 1.05$. However, we point out that the long wavelength mode can not be observed physically, because its growth rate is always less than the short wavelength mode. The long wavelength mode resembles that of the so-called mode A in the drag wake of a fixed bluff body. While the short wavelength mode repeats the perturbation field every second cycle of the base flow.

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Keywords: Flapping foil; Propulsive wake; Linear instability; Floquet analysis

1. Introduction

It is well-known that the flow behind a circular cylinder undergoes transition to three-dimensional flow and eventually turbulence through a sequence of mode emergence[1]. When the Reynolds number (*Re*) exceeds 190, the wake transits to 'Mode A' with a spanwise wavelength of 3-4 cylinder diameters. A further transition to 'Mode B' occurs as *Re* is increased to 230 − 250 with a shorter spanwise wavelength of approximately one diameter. Another theoretically possible mode is the quasi-periodic 'mode QP' [2]. Investigations have also been conducted on the flow fields around bluff bodies with non-circular sections [3]. For a square cylinder, 'Mode S' was found to be critical within $150 < Re < 225$, but only after the other modes had already undergone transition, and therefore may not be observed experimentally. Mode S was firstly found to be subharmonic [4], with a period double that of the base flow. It was then discovered that Mode S has a complex Floquet multiplier with real part negative and a small imaginary component, and thus it repeats every second cycle [2].

Most previous studies focused on drag wakes or the classic BvK vortex street. It is unknown whether a similar transition route exists in a thrust wake. As a well-known thrust wake, the wake behind a flapping airfoil has attracted much

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attention [5,6] because it acts as a simplified model of an aquatic propulsor [7,8]. The majority of these studies were based on a two-dimensional hypothesis, under the assumption that three-dimensional effects play a small role which may be neglected. When three-dimensionality was considered, it was typically in the context of airfoils with finite spans [9–11]. In such cases, instead of the inherent breaking of the spanwise reflection symmetry, three-dimensional flow is enforced on the flow wake by wing tips.

However, it was noted in the experiments on a pitching foil [12,13] that when the reverse BvK street is deflected 'the quasi-two-dimensional nature of the flow is rapidly lost and the coherence of vortical structures in the horizontal midplane where the observations are performed does not prevail for more than two cycles' [12]. Though they conjectured that the finite span of the airfoil should be responsible for the 3D effects, it is still worthwhile to investigate if an inherent three-dimensional transition in the wake occurs. As we have recently reported that tip effects are limited with little influence on the flow at the middle of the span for a flapping wing with a high-aspect-ratio [14]. Here, as a first step to understand this complicated physical process, we investigate in isolation 'inherent 3D transition' by considering the evolution of three-dimensional disturbances in a two-dimensional base flow.

2. Problem formulation and numerical methodology

We consider a NACA0015 airfoil pitching according to a sinusoid profile, with the pivoting point located at the leading edge. The amplitude of pitching angle is denoted by θ_0, and the peak-to-peak amplitude of the trailing edge is denoted by A. We define the Reynolds number $Re = Uc/\nu$, where c is the chord length, and ν is the kinematic viscosity. We choose $Re = 1500$ for most our simulations, corresponding to $Re_D = 225$ ($Re_D = UD/\nu$, where D is the thickness of the airfoil), which is close to that chosen in Ref.[12]. The two flapping parameters which characterize the wake dynamics are the appropriately scaled amplitude A_D, defined as $A_D = A/D$, and the Strouhal number S_r, defined as $Sr = fD/U$.

The base flow is simulated using a finite-volume based code. The Floquet stability analysis is applied to study the stability of a two-dimensional time-periodic base flow to three-dimensional disturbances. By introducing a perturbation spanwise wavenumber $\beta = 2\pi/\lambda_z$, where λ_z is the spanwise wavelength of the disturbance, we can approximate the perturbation velocity and pressure as a sum of Fourier modes in the spanwise direction. According to Floquet theory, only the dominant Floquet mode remains after many cycles. We measure the Floquet multiplier $|\mu|$ as the ratio of the current L_2 norm of any of the velocity perturbations to the L_2 norm exactly one period prior. If $|\mu| > 1$, the corresponding perturbation field grows exponentially from one period to the next and hence the base flow is linearly unstable to perturbations of the selected spatial wavelength in the z-direction.

3. Results and discussions

The wake undergoes two transitions for the base flow. The first transition changes the wake from a classic BvK vortex street into a reverse BvK vortex street, which results in a net propulsive force. It retains the Z_2 spatio-temporal symmetry. Above a certain threshold, the second transition induces an asymmetric pattern in the wake, breaking the Z_2 symmetry, with the consequence that the net force generated by the flapping motion is not aligned with the foil symmetry plane. Equivalently, a mean lift force accompanies the production of thrust.

Instead of performing a systematic parametric study in (A_D, Sr) space, we select a typical Sr number, $Sr = 0.22$, and demonstrate the three qualitatively different scenarios that the wake of a flapping foil undergoes. They are classic BvK vortex streets as shown in Fig.1(a), reverse BvK vortex streets as shown in Fig.1(b), and deflected vortex streets as shown in Fig.1(c,d). For $A_D = 2.0$, (Fig.1(c)) the reverse BvK propulsive vortex street departs slightly from the centre line, while for $A_D = 2.13$ (Fig.1(d)) an apparent wake deflection occurs with a strong dipolar structure propagating obliquely to one side of the symmetry line in each flapping cycle, while a much weaker single vortex is shed on the other side. This transition of the wake can be quantitatively studied by examining the mean flow in the wake. As shown in Fig.2, when $A_D = 2.0$ the jet profile slightly deflects from the symmetry line. As the amplitude is increased further, the jet profile shifts further from the symmetry line, and a velocity deficit appears on the other side of the symmetry line. The deficit of the wake velocity, and so the deficit of momentum, reduces the net thrust acting on the flapping foil. In the range of $A_D = 2.13 - 2.80$, the amplitude of the wake jet varies only slightly with

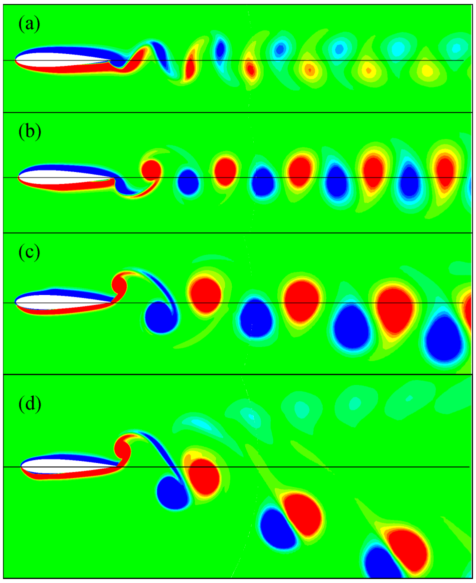

Fig. 1. Vorticity contours in the wake at Re = 1500 with nondimensional amplitude: (a) A_D = 0.36, (b) A_D = 1.07, (c) A_D = 2.0, (d) A_D = 2.13.

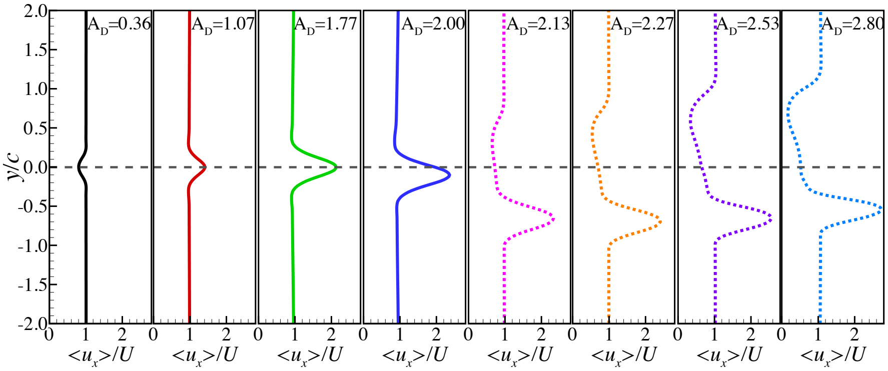

Fig. 2. Streamwise component of the time-averaged velocity $\langle u_x \rangle$, measured at a location three chords downstream from the leading edge.

increasing flapping amplitude, while the amplitude of the wake deficit increases greatly, implying a degradation of the propulsive efficiency.

In Fig.3 we plot the variation with spanwise wave number β of the magnitude of the largest Floquet multiplier $|\mu|$. For Re = 1500, when $A_D \leq 2.53$, the largest Floquet multiplier over the entire range of β is less than one, indicating that the flow is stable to all infinitesimal three-dimensional perturbations. For Re = 1500 and A_D = 2.8, there is a range of wave numbers for which $|\mu| > 1$, indicating that the two-dimensional base flow is unstable to some linear three-dimensional disturbances. The wave number with the maximum growth rate is $\beta = 30$ and it corresponds to a nondimensional spanwise wavelength of $\lambda_z = 0.21$, nondimensionalized by the chord length c. A second local maximum can be observed at $\beta = 6$ ($\lambda_z = 1.05$). To determine the critical Reynolds number for each mode, we vary the Reynolds number, and find that when Re = 600, the short wavelength mode is just becoming marginal, but the long wavelength mode is below the neutral line. This indicates that the long wavelength mode is never dominant, and thus is never expected to be physically observed.

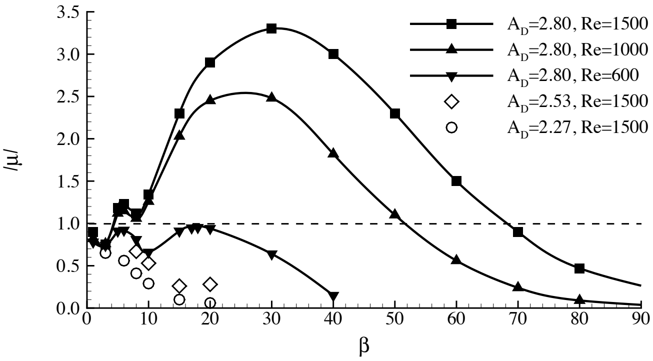

Fig. 3. Variation of Floquet multiplier magnitude $|\mu|$ with wavenumber β. The line $|\mu|$=1 corresponds to neutral stability of the Floquet modes.

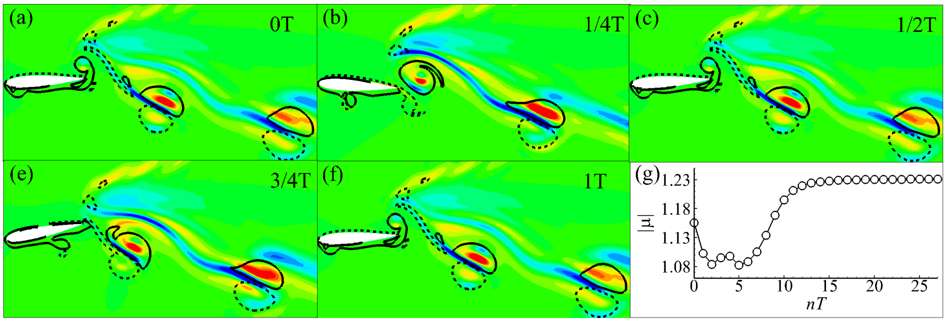

Fig. 4. (a)-(f) Instantaneous streamwise perturbation vorticity for $Re = 1500$, $A_D = 2.8$ and $\beta = 6.0$. Blue and red colors denote negative values and positive values respectively. Spanwise vorticity contours of the base flow are also shown with solid and dashed lines for positive and negative values respectively. (g) Time variation of $|\mu|$.

To illustrate the characteristics of these two modes, we present the perturbation flow fields for the long wavelength mode and the short wavelength mode in Fig.4 and Fig.5 respectively. The long wavelength mode resembles the mode A instability of the cylinder wake, with perturbation vorticity localized within the base flow vortices. Fig.4(g) shows the convergence of $|\mu|$, indicating that the dominant mode is isolated in the perturbation field 15 cycles after it is initialized. The constant $|\mu|$ means that the mode contains no imaginary component, implying a synchronous bifurcation. The short wavelength mode originates in the region between the forming vortices in the near wake. As seen in Fig.5(c), during the downward pitching of the trailing edge, the perturbation grows on the edge of the newly forming vortex. Then, as the trailing edge pitches up (Fig.5(d)), a vortex with opposite sign emerges below the previous one, and so a dipolar structure emerges. As the dipole propagates obliquely downstream, the perturbation is stretched out and transferred to the emerging dipole of the next cycle (see Fig.5g). It is apparent that two successively shed dipoles contain perturbation fields with opposite signs, as does the ribbon region behind the dipole. This period-doubling behavior implies that this mode might be subharmonic, which is also supported by its physical realizability, since it has the largest growth rate among all wave numbers. However, the fluctuation of $|\mu|$ with time apparent in Fig.5(j) indicates the existence of an imaginary component of the Floquet multiplier. Therefore, there is still not enough evidence that this mode should be classified as either a quasi-periodic mode or as a subharmonic mode.

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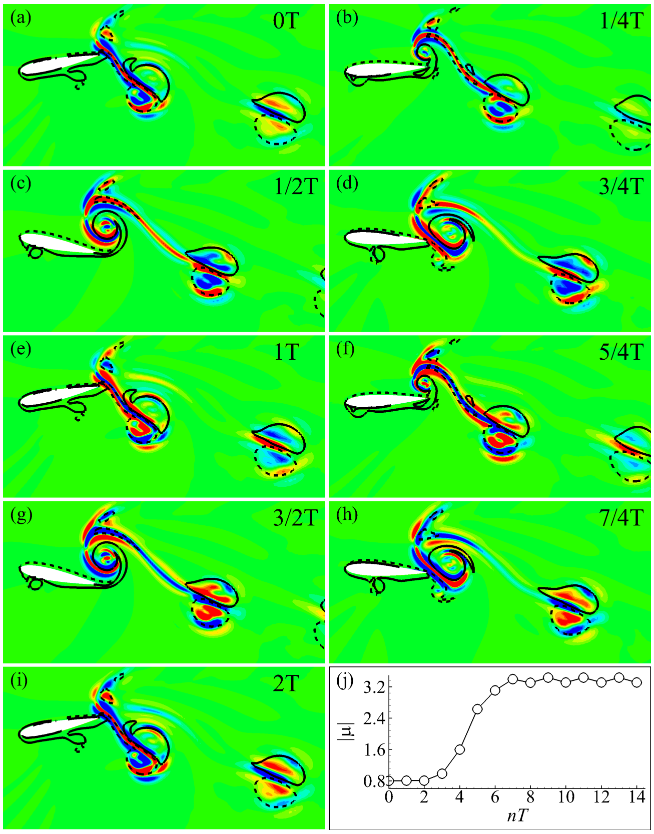

Fig. 5. (a)-(f) Instantaneous streamwise perturbation vorticity for $Re = 1500$, $A_D = 2.8$ and $\beta = 30.0$. Blue and red colors denote negative values and positive values respectively. Spanwise vorticity contours of the base flow are also shown with solid and dashed lines for positive and negative values respectively. (g) Time variation of $|\mu|$.

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